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***Abstract*—In this paper, we study the interconnection of two robots, which are modeled as port-controlled Hamiltonian systems through a transmission line with time delay. There will be no analysis of the time delay, but its presence justifies the use of scattering variables to preserve passivity. The contributions of the paper are twofold: first, a geometrical, multidimensional, power-consistent exposition of telemanipulation of intrinsically passive controlled physical systems, with a clarification on impedance matching, and second, a system theoretic condition for the adaptation of a general port-controlled Hamiltonian system with dissipation (port-Hamiltonian system) to a transmission line.**

***Index Terms*—Hamiltonian systems, passivity, physical control, scattering, telemanipulation.**

I. Introduction

AMONG THE MORE relevant aspects of telemanipulation, an interesting control problem is given by the presence of a nonnegligible time delay present in the transmission line between the "master" and the "slave" devices. The time delay may introduce instability effects in the overall control loop, especially when force information is exchanged between master and slave, and performances have to be improved.

Several approaches have been proposed in the literature in order to deal with this problem of time delays. Among the first contributions see, e.g., [1], where a one-dimensional case has been considered. Later, an extension has been presented in [5], where important considerations on the line causality and extensions with adaptation techniques are treated. For an overview and comparison on the control techniques presented in the literature, one may refer to [6]–[8].

In [4], a geometrical multidimensional case is presented, which uses digital transmission of data in order to create a perfectly bilateral telemanipulation system on a transmission line with varying, nonneglectable delays: the Internet.

The results presented in [4] are extended in this paper, and a general setting for telemanipulation of port-Hamiltonian systems is presented. A new system theoretic condition is introduced which can be used to test whether proper matching is taking place, and a possible measure of matching is also introduced. The use of port-Hamiltonian systems is extremely useful because it allows, in a single framework, a general, geometric description of any physical system of any dimension using a nicely defined network structure. This allows us to elegantly study telemanipulation of any multidimensional system in a coordinate-free way.

It is important to stress that the main advantage of this paper is the transmission of vector quantities as geometrical tensors and not as an array of scalar components. This feature has important invariance and intrinsic properties. The time delay is not named explicitly, but it has been already shown in the literature [1], [5] that a delay line can be modeled as a two-port using scattering.

The paper is organized as follows. Section II introduces the basic idea concerning geometric scattering which is needed in order to use scattering techniques for multidimensional tensors, Section III addresses the choice of the causality which is made for the interconnection between the controller and the transmission line (i.e., effort in and flow out or vice versa), Section IV reviews known issues concerning the modification of the impedance seen through a system's power port and extends the results to the multidimensional case, Section V gives a simple, but very useful, system theoretic condition which can be used to study the impedance matching of multidimensional system coupling, Section VI stresses the novelty of the presented theory by means of a meaningful geometrical example, and Section VII draws some conclusions and introduces new research directions.

II. Geometric Scattering

Scattering variables are well known in network theory [9], [10], and in control [1], [5]. To the best of the authors' knowledge, the first works which present scattering variables from a geometric point of view are [4] and [11]–[13]. This allows us to implement a telemanipulating system from an intrinsically geometric point of view, which means transmitting not coordinates but vectors, i.e., screws and twists corresponding to spatial motion for mechanical systems. The presented framework is coordinate free, and this is an important feature when vectorial quantities are analyzed. It means that the achieved results are "intrinsic" in the sense that they do not depend on the specific coordinates chosen to implement them.

The main idea is as follows. Given a vector space $\mathcal{V}$, we can consider the vector space

$$\mathcal{D} := \mathcal{V} \times \mathcal{V}^*$$

where $\mathcal{V}^*$ indicates the dual vector space of $\mathcal{V}$. On $\mathcal{D}$ there exists a canonical, symmetric, two-covariant tensor called $+$ *pairing*.

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This symmetric, nondegenerate 2_form is defined by the bilinear operation

$$\langle (f_1, e_1), (f_2, e_2) \rangle_+ := \langle e_1, f_2 \rangle + \langle e_2, f_1 \rangle$$

where $(f_i, e_i) \in \mathcal{D}$ and $\langle e_i, f_j \rangle$ denote the intrinsic dual pairing. Using this tensor, it is also possible to give a geometric definition of a Dirac structure as a "self-orthogonal" subspace for the $+$ pairing [11], [13], [14]. We can define B as a matrix corresponding to a choice of a basis of $\mathcal{V}$. e_i are directly the basis vectors

$$B := \begin{pmatrix} e_1 & \dots & e_n \end{pmatrix}$$

and the dual base as the columns of a matrix B_*

$$B_* := \begin{pmatrix} e_1^* & \dots & e_n^* \end{pmatrix}$$

such that $B_*^T B = B^T B_* = I$. We can then define the corresponding base matrix for $\mathcal{D}$ as the columns of a matrix $\bar{B}$

$$\bar{B} := \begin{pmatrix} B & 0 \\ 0 & B_* \end{pmatrix}$$

and, eventually, the adjoint matrix $\bar{B}_*$

$$\bar{B}_* := \begin{pmatrix} B_* & 0 \\ 0 & B \end{pmatrix}.$$

A representation of the $+$ pairing is then

$$T_{ij} = \bar{B}_* \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \bar{B}_*^T \tag{1}$$

where the indexes indicate it is a tensor of type $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$. In order to define subspaces in a coordinate-free way, we shall use a metric on $\mathcal{V}$ which corresponds to a characteristic impedance $B_* Z B_*^T$. We can define a two-contravariant tensor on $\mathcal{D}$ based on Z as

$$Y^{li} = \bar{B} \begin{pmatrix} Z^{-1} & 0 \\ 0 & Z \end{pmatrix} \bar{B}^T \tag{2}$$

and then consider the eigenvalues of

$$L_j^l := Y^{li} T_{ij} = \bar{B} \begin{pmatrix} 0 & Z^{-1} \\ Z & 0 \end{pmatrix} \bar{B}_*^T. \tag{3}$$

λ is an eigenvalue of L_j^l if there exist nonzero (e, f) such that

$$\lambda \begin{pmatrix} f \\ e \end{pmatrix} = \begin{pmatrix} 0 & Z^{-1} \\ Z & 0 \end{pmatrix} \begin{pmatrix} f \\ e \end{pmatrix} \tag{4}$$

which implies that it should be

$$\begin{cases} \lambda f = Z^{-1} e \\ \lambda e = Z f \end{cases} \Rightarrow \begin{cases} \lambda^2 f = Z^{-1} Z f = f \\ \lambda^2 e = Z Z^{-1} e = e \end{cases}$$

from which it follows that the eigenvalues are $\lambda = \pm 1$. We can, therefore, define two eigensubspaces associated to the eigenvalue $+1$ and -1, respectively, which turn out to be of the same dimension as $\mathcal{V}$ and dependent on Z. We denote this as

$$\mathcal{D} = \mathcal{S}_Z^+ \oplus \mathcal{S}_Z^- \tag{5}$$

which implies that for each Z there is a unique way to express a power pair $(f, e) \in \mathcal{D}$ as the sum of two elements, $s^+ \in \mathcal{S}_Z^+$ and $s^- \in \mathcal{S}_Z^-$. Furthermore, it is possible to see that an expression of the eigenspaces corresponding to the eigenvalues ± 1 can be given in both a kernel or image form as follows:

$$\mathcal{S}_Z^+ = \ker \begin{pmatrix} I & -Z^{-1} \end{pmatrix} \bar{B}_*^T = \operatorname{Im} \bar{B} \begin{pmatrix} Z^{-1} \\ I \end{pmatrix} \frac{N}{\sqrt{2}} \tag{6}$$

and

$$\mathcal{S}_Z^- = \ker \begin{pmatrix} Z & I \end{pmatrix} \bar{B}_*^T = \operatorname{Im} \bar{B} \begin{pmatrix} -I \\ Z \end{pmatrix} \frac{N^{-1}}{\sqrt{2}} \tag{7}$$

where N is the symmetric square root of Z ($Z = NN$), which always exists under the hypothesis that Z is symmetric and positive semidefinite (see *Theorem 2*). The last terms of (6) and (7) (including the square roots) are used for normalization, such that the columns of the matrices whose image is considered are orthonormal in the norms induced by the $+$ pairing as shown in (9) and (11).

It is now possible to check whether $\mathcal{S}_Z^-$ and $\mathcal{S}_Z^+$ are orthogonal for the $+$ pairing. Using the image representations, it is possible to see that this is the case iff the following matrix is identically zero:

$$\begin{aligned} &\frac{1}{2} N^{-T} \begin{pmatrix} -I & Z^T \end{pmatrix} \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} Z^{-1} \\ I \end{pmatrix} N \\ &\qquad = \frac{1}{2} N^{-T} (Z^T Z^{-1} - I) N \end{aligned} \tag{8}$$

which is the case iff the tensor Z is symmetric. We will shortly see that this condition is also essential to achieve the power decomposition which is fundamental for the scattering representation.

By restricting the $+$ pairing to $\mathcal{S}_Z^+$, we obtain an inner product on $\mathcal{S}_Z^+$, and by restricting it to $\mathcal{S}_Z^-$ we obtain an inner product on $\mathcal{S}_Z^-$. Once again, using the image representation of $\mathcal{S}_Z^+$, it is possible to see that the induced inner product on $\mathcal{S}_Z^+$, using as base of $\mathcal{S}_Z^+$, the columns of

$$S_Z^+ := \bar{B} \begin{pmatrix} Z^{-1} \\ I \end{pmatrix} \frac{N}{\sqrt{2}} \tag{9}$$

are

$$\begin{aligned} &\frac{1}{2} N^T \begin{pmatrix} Z^{-T} & I \end{pmatrix} \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} Z^{-1} \\ I \end{pmatrix} N \\ &= \frac{1}{2} N (Z^{-T} + Z^{-1}) N \\ &= N \operatorname{sym}(Z^{-1}) N = I. \end{aligned} \tag{10}$$

This implies that the chosen base S_Z^+ is orthonormal in the induced inner product. Similarly, using the image representation of $\mathcal{S}_Z^-$, it is possible to see that the induced inner product on $\mathcal{S}_Z^-$, using as base for $\mathcal{S}_Z^-$, the columns of

$$S_Z^- := \bar{B} \begin{pmatrix} -I \\ Z \end{pmatrix} \frac{N^{-1}}{\sqrt{2}} \tag{11}$$

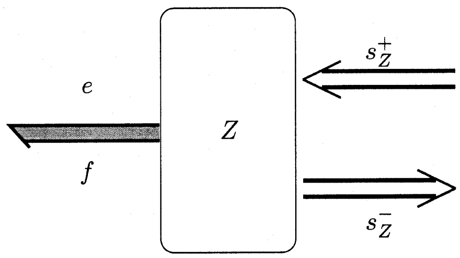

Fig. 1. Scattering transformation.

are

$$\begin{aligned}\frac{1}{2}N^{-T}\begin{pmatrix}-I & Z^T\end{pmatrix}\begin{pmatrix}0 & I\\ I & 0\end{pmatrix}\begin{pmatrix}-I\\ Z\end{pmatrix}N^{-1}\\ = -\frac{1}{2}N^{-1}(Z+Z^T)N^{-1}\\ = -N^{-1}\mathrm{sym}(Z)N^{-1} = -I.\end{aligned} \tag{12}$$

Since the previous matrix is $-I$, we will use it as induced product, minus the result of (12). Once again, this shows that the chosen base is orthonormal for the induced inner product. It is now possible to present the most important result regarding geometric scattering, which was also reported in [11] and [13] under less general conditions.

Theorem 1 (Scattering Power Decomposition): Given any $(f, e) \in \mathcal{D}$ and any positive definite, symmetric, two-covariant tensor Z, the following relation holds:

$$\langle e, f\rangle = \frac{1}{2}\left\|s_Z^+\right\|_+^2 - \frac{1}{2}\left\|s_Z^-\right\|_-^2$$

where $s_Z^+ \in \mathcal{S}_Z^+$, $s_Z^- \in \mathcal{S}_Z^-$, $(f, e) = s_Z^+ + s_Z^-$, and $\| \ \|_+$ and $\| \ \|_-$ are, respectively, the induced inner products on $\mathcal{S}_Z^+$ and $\mathcal{S}_Z^-$.

Proof: Due to (5), we are not restricting ourselves if we consider

$$\begin{pmatrix}f\\ e\end{pmatrix} = \frac{\bar{B}}{\sqrt{2}}\left(\begin{pmatrix}Z^{-1}\\ I\end{pmatrix}Ns_+ + \begin{pmatrix}-I\\ Z\end{pmatrix}N^{-1}s_-\right). \tag{13}$$

In this case, we have

$$\langle e, f\rangle = \frac{1}{2}\left(s_+^T s_+ - s_- s_- - s_+^T s_- + s_-^T s_+\right) \tag{14}$$

which directly proves the result using (10) and the negative of (12). ■

Remark 1: It is now evident why the scattering subspaces should be orthogonal using the $+$ pairing. Only under this condition do we obtain the previous decomposition, which is fundamental because it shows that we can algebraically write the power flow as the sum of a positive and negative power depending only on the two scattering variables. This can be interpreted as power going in opposite directions as shown in Fig. 1 where it is shown in bond graph notation that the power bond has indeed the same direction as the variables s_Z^+ due to the accordance of sign. Intuitively, s_Z^+ can be thought of as a wave transporting power in the direction of the bond, and s_Z^- transporting power in the opposite direction.

A. Dimension of the Space of Scattering Decompositions

Due to the fact that Z should be symmetric, and due to their representations, it is possible to see that there are an infinite number of possible scattering decompositions which are parameterized by the symmetric tensors Z (which constitute a space of dimension $n(n+1)/2$).

B. Plus-Product Invariance

The set of all possible scattering subspaces could be also defined using the possible changes of coordinates which would leave the $+$ pairing invariant, or equally, the possible transformation of subspaces, which would leave the $+$ pairing invariant.

C. Scattering Decomposition

Using (13) in the chosen base, the mapping-relating efforts and flows to scattering variables has the numerical representation [1]

$$\begin{cases} f = \frac{N^{-1}}{\sqrt{2}}(s_+ - s_-)\\ e = \frac{N}{\sqrt{2}}(s_+ + s_-)\end{cases} \tag{15}$$

and inverting the relations

$$\begin{cases} s_+ = \frac{N^{-1}}{\sqrt{2}}(e + Zf)\\ s_- = \frac{N^{-1}}{\sqrt{2}}(e - Zf).\end{cases} \tag{16}$$

It can be seen that this is a generalization of the scattering decomposition for scalar power conjugate variables as is well known in circuit theory, and as used in [1] and [5], but it is important to realize that for scalar quantities, no geometry and coordinate invariance issues are relevant. In the multidimensional presented case instead, coordinate invariance plays a role, and it has been shown in Section II that this formulation is well posed and coordinate invariant. This is a consequence of the coordinate invariance decomposition reported in (5), which shows in a geometrical way that any pair of power conjugate variables describing a power port can be represented, after having chosen a metric Z, in a *unique* way as the sum of two elements, the wave variables which are orthogonal in the sense of the $+$ pairing.

III. Causality and Sign Issues

It has been shown in [1] for the simpler scalar (siso) case that in order to preserve passivity with a transmission line connecting two systems, the power port connected to the transmission line can be coded and decoded in scattering variables. The coded signal s_Z^- can be sent on the line, and it will be used by the other side as the incoming signal s_Z^+. The total energy stored on the line is, therefore, the integral of the traveling signal. Since the variable s_Z^+ is always an input for the two systems attached to the line, we have two causal possibilities:

- computing e and s_Z^- as a function of f and the incoming wave s_Z^+;
- computing f and s_Z^- as a function of e and the incoming wave variable s_Z^+.

[1]The matrix $\bar{B}$ has disappeared since we are using now a numerical representation in this base.

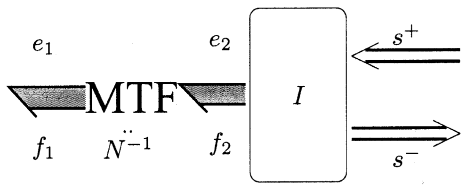

Fig. 2. Scaling of impedance.

It has been shown in [5] that there are multiple reasons for choosing the second option. Furthermore, if we want to have a perfectly symmetric system, the causalities at both sides should be the same. A first possibility, from a purely causal point of view, would be to let the line behave in a gyrative action in such a way that for the line length and delay tending to zero, the effort supplied by one side would become the input flow of the other, and vice versa. Unfortunately, such a system cannot work because in steady state, when the master and slave do not move, the velocities should be zero ($f = 0$), but at the same time, we want that a reflection of a force different from zero could take place if necessary (that is, $e \neq 0$). Since with a connection called symplectic [2] f and e are equal, this is not possible. This implies that the systems at both sides should have an *impedance* causality and that the line in the limit of its length tending to zero should *not* behave as a gyrative action.

From this, we conclude we have to choose exactly the same scattering mapping on both sides and connect the departing wave of one side to the incoming wave of the other side. This has an important consequence. If the line length and its delay are tending to zero, then we get a causal inconsistency since the line should supply the same power variable at both sides, namely, the flow f. This would correspond to an algebraic loop with no delays, and implies that the energy storage of a finite-length line "fixes" the causal problem exactly as a mass would do to connect two physical systems through springs. In other words, the causality problem is solved using an energy-storing two-port (transmission line) versus the gyrator.

IV. Line Impedance Adaptation

The case in which $Z = I$ corresponds to an impedance which maps the base B to its intrinsic dual B_*. In this case, and for the chosen coordinates, it is possible to see that I is indeed the impedance felt by the system attached to the power port when we suppose no power coming from the line ($s^+ = 0$). Looking at another causal form of (16) for $Z = I$

$$e = \sqrt{2}s^+ - f \rightarrow e = -f$$

which corresponds to an identity impedance in the chosen coordinates. The general impedance decomposition reported in (16) can also be interpreted in a different way using what is called in bond graphs a *transformer*. The resulting scheme is given in Fig. 2. The equation characterizing a transformer with matrix transformation N^{-1} is

$$\begin{aligned} f_1 &= N^{-1} f_2 \\ e_2 &= N^{-T} e_1 \end{aligned} \tag{17}$$

where, in our case, N is a square, in general time-varying, nonsingular matrix. The impedance seen at the (e_1, f_1) port is the matrix Z_1 such that $e_1 = Z_1 f_1$ and substituting the transformer equations

$$e_1 = N^T e_2 = -N^T f_2 = -\left(N^T N\right) f_1$$

which implies that $Z_1 = N^T N$. Note that the equality $e_2 = -f_2$ is a consequence of the fact that for hypothesis, the impedance before the transformer is the identity as shown also in Fig. 2. The latter result is a trivial, well-known result in network theory. Z_1 is a positive definite, symmetric, two-contravariant tensor, and therefore

$$P = e_1^T f_1 = -f_1^T Z_1^T f_1 \leq 0 \tag{18}$$

which indicates that the power is going in the opposite direction as indicated by the power bond in Fig. 2, and therefore, toward the transmission line as expected. A question arises: is it possible to find an $\mathbb{MTF}$ of Fig. 2 such that the impedance seen from (e_1, f_1) can get any symmetric desired value Z?

The answer is given by the following trivial linear algebra result:

Theorem 2: Given any symmetric, positive semidefinite matrix Z, there exists always a symmetric matrix N, such that

$$Z = N^T N = N^2.$$

With this result, we can state that *all* meaningful impedances (symmetric and positive definite) Z can be generated by a proper choice of a transformer N^{-1}.

This implies that since the identity-scattering transformation can be expressed as found previously by

$$\begin{cases} s^+ = \frac{1}{\sqrt{2}}(e_2 + f_2) \\ s^- = \frac{1}{\sqrt{2}}(e_2 - f_2) \end{cases}$$

and using (17)

$$\begin{cases} s^+ = \frac{1}{\sqrt{2}}(N^{-1} e_1 + N f_1) \\ s^- = \frac{1}{\sqrt{2}}(N^{-1} e_1 - N f_1). \end{cases}$$

Eventually, we obtain the scattering transformation for a generic, multidimensional impedance Z, which corresponds indeed to (16)

$$\begin{cases} s^+ = \frac{N^{-1}}{\sqrt{2}}(e_1 + Z f_1) \\ s^- = \frac{N^{-1}}{\sqrt{2}}(e_1 - Z f_1). \end{cases} \tag{19}$$

As already seen, Z is a fundamental parameter for the line, which characterizes the wave variables s^+, s^-, and directly effects the system behavior.

It is important to know that, in a real analog transmission line like a coaxial cable or a twisted pair, the impedance is obviously a physical characteristic of the line which we cannot influence. On the contrary, in a digital transmission line, only data are sent, and the scattering mapping of Fig. 2 corresponds to an algorithmic implementation which codes and decodes the sent and received data. Future work will formally analyze the correctness

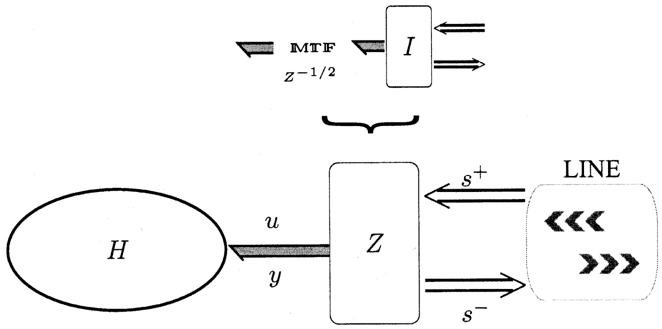

Fig. 3. Power interconnection with a Hamiltonian system.

of this analogy by relating the continuous time case to a discrete one.

V. Impedance Matching

Impedance matching is a well-known problem in transmission lines. The energy received from the line has to be absorbed by master and slave systems. Once the impedance Z seen at the power port of Fig. 3 is chosen (model of the line), a system with the same impedance needs to be connected at the end of the line to avoid wave reflections. This guarantees continuity of impedance with respect to the line.

A general system theoretical condition for matching of a general physical system connected to a line as in Fig. 3 can now be simply postulated as follows.

Principle 1 (Matching Condition): The system seen at the scattering side of the transformation of Fig. 3 and having s^+ as input and s^- as output has to be of relative degree ≥ 1 (that is, the system should have no direct feed through).

This implies that there should not be an algebraic relation between the waves s^+ and s^-, which is exactly equivalent to the idea of undiscriminated reflection of power. In intuitive terms, the power should be first somehow "processed" by the master (resp. slave) before some information is sent back to the slave (resp. master).

Now, we want to investigate what conditions *Principle 1* imposes on a generic port-Hamiltonian system [2] (which could represent any physical explicit system), connected at the end of the line as in Fig. 3. Since we consider port-controlled generalized Hamiltonian systems (both master and slave sides), we have

$$\begin{cases} \dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + G(x)f \\ e = G^T(x)\frac{\partial H(x)}{\partial x}. \end{cases} \tag{20}$$

From the scattering transformation of (19), we can obtain the port variables as functions of the wave variables

$$s^+ + s^- = \sqrt{2}N^{-1}e \Rightarrow e = \frac{1}{\sqrt{2}}N(s^+ + s^-)$$
$$s^+ - s^- = \sqrt{2}Nf \Rightarrow f = \frac{1}{\sqrt{2}}N^{-1}(s^+ - s^-)$$

and thus the Hamiltonian system of (20) is transformed to

$$\begin{cases} \dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + \frac{1}{\sqrt{2}}G(x)N^{-1}(s^+ - s^-) \\ e = G^T(x)\frac{\partial H(x)}{\partial x} = \frac{1}{\sqrt{2}}N(s^+ + s^-). \end{cases}$$

The new system having as input s^+ and as output s^- is given as

$$\begin{cases} \dot{x} = (J(x) - R(x) - G(x)N^{-1}N^{-1}G^T(x))\frac{\partial H(x)}{\partial x} \\ \quad +\sqrt{2}N^{-1}G(x)s^+ \\ s^- = \sqrt{2}N^{-1}G^T(x)\frac{\partial H(x)}{\partial x} - s^+. \end{cases}$$

Hence, we conclude that the input s^+ is directly fed through to the output s^-. This implies that any power arriving from the line is sent back independently of the state of the system connected to the line. Thus, the Hamiltonian system of (20) does not satisfy *Principle 1*, and is not general enough for impedance matching.

Hence, in order to meet *Principle 1*, we have to enlarge the class of port-Hamiltonian systems. We do this by considering port-Hamiltonian systems of the extended form

$$\begin{cases} \dot{x} = (J(x) - R(x))\frac{\partial H(x)}{\partial x} + G(x)f \\ e = G^T(x)\frac{\partial H(x)}{\partial x} + B(x)f \end{cases} \tag{21}$$

with $B(x) \geq 0$ a newly added dissipation matrix.

In this case, we obtain, using (19) and (21), the new output equation

$$s^- = \left(\frac{B(x)}{\sqrt{2}}N^{-1} + \frac{1}{\sqrt{2}}N\right)^{-1} G^T(x)\frac{\partial H(x)}{\partial x} + Fs^+ \tag{22}$$

where

$$F = \left(B(x)N^{-1} + N\right)^{-1}\left(B(x)N^{-1} - N\right)$$

which implies, using *Principle 1*, that to have impedance matching we must have

$$B = NN = Z$$

since this implies $F = 0$.

Thus, a system of the form of (21), for which $B(x)$ is equal to the impedance Z of the line to which it is connected, guarantees the matching condition expressed in *Principle 1*, and eliminates any indiscriminate reflection of power.

Using the previous setting, it is even possible to give a coordinate invariant measure of the level of matching if this is not perfect. This can be easily done considering the induced norm of F. From (22) it is possible to see that F is a mapping of the following form:

$$F : \mathcal{S}^+ \to \mathcal{S}^- \; ; \; s^+ \mapsto Fs^+$$

and since $\mathcal{S}^+$ and $\mathcal{S}^-$ are normed spaces, we can define the following induced norm for F:

$$\|F\| := \sup_{s \neq 0} \frac{\|Fs\|_-}{\|s\|_+}.$$

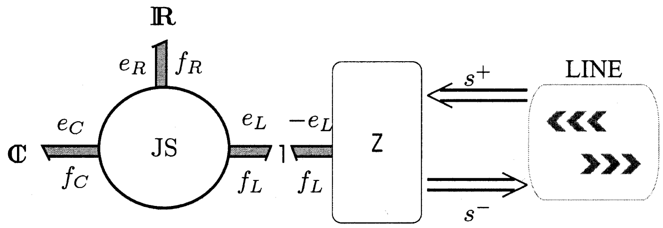

Fig. 4. Interconnection description.

If this norm is zero, perfect matching is obtained; if this norm is one, perfect reflection takes place.

A. An Interconnection Approach

The previous conclusion can be obtained similarly using the framework of Dirac structures [15]–[18]. The advantage of using this framework is that it makes more clear that the adaptation condition is only dependent on the network structure (interconnection) and the resistive part of the system connected to the transmission line. For simplicity, we consider a network structure with reference to Fig. 4 relating three ports: the power port connected to the line (f_L, e_L), the power port connected to a dissipating component (f_R, e_R), and a power port connected to a storage element (f_C, e_C). The network structure can be expressed by a linear mapping of the form

$$\begin{pmatrix} e_L \\ e_C \\ e_R \end{pmatrix} = \begin{pmatrix} D_L & G_1 & G_2 \\ -G_1^T & D_C & G_3 \\ -G_2^T & -G_3^T & D_R \end{pmatrix} \begin{pmatrix} f_L \\ f_C \\ f_R \end{pmatrix}$$

where D_L, D_C, and D_R are skew symmetric. A dissipating element of the system has characteristic equations of the form $e_R = Rf_R$ with R symmetric and positive semidefinite. This implies that

$$f_R = (D_R - R)^{-1} G_2^T f_L + (D_R - R)^{-1} G_3^T f_C$$

and, therefore

$$\begin{pmatrix} e_L \\ e_C \end{pmatrix} = \begin{pmatrix} B & A \\ C & D \end{pmatrix} \begin{pmatrix} f_L \\ f_C \end{pmatrix}$$

where

$$B := D_L + G_2(D_R - R)^{-1} G_2^T \tag{23}$$
$$A := G_1 + G_2(D_R - R)^{-1} G_3^T \tag{24}$$
$$C := -G_1^T + G_3(D_R - R)^{-1} G_2^T \tag{25}$$
$$D := D_C + G_3(D_r - R)^{-1} G_3^T. \tag{26}$$

Applying the scattering transformation to the power port $(f_L, -e_L)$, it is possible to obtain

$$s^- = -(N + N^{-1}B)(N - N^{-1}B)^{-1} s^+ + K f_c$$

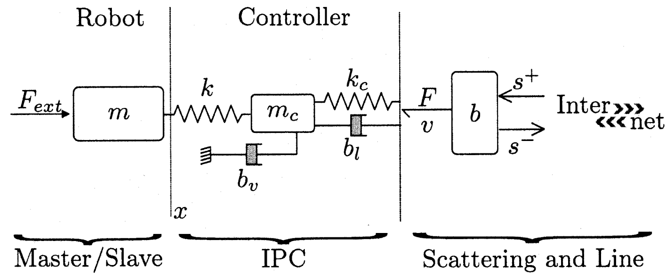

Fig. 5. One-dimensional teleoperator.

where

$$K = -\frac{1}{\sqrt{2}}(N + N^{-1}B)(N - N^{-1}B)^{-1} N^{-1} A - N^{-1} A.$$

This implies for *Principle 1* that for adaptation we need to have $(N + N^{-1}B) = 0$ and, therefore, $B = -Z$, which implies

$$D_L + G_2(D_R - R)^{-1} G_2^T = -Z$$

with Z symmetric. This implies that necessarily $D_L = 0$, and furthermore, if we suppose G_2 to be square and nonsingular, that also $D_R = 0$, implying that

$$R^{-1} = G_2^{-1} Z G_2^{-T}.$$

From the previous analysis, we can conclude that the adaptation is independent of the state of the system, and only depending on the system interconnection and its dissipative term.

B. Damping Injection in Telemanipulation

As an example of the feed-through term, let's consider a one-dimensional "robot" composed of a single mass. It was shown in [19] that to control its interactive behavior by using only position measurements, it is possible to use the intrinsically passive controller (IPC) reported in Fig. 5, where the supervision power port (F, v) was connected to a supervisory controller. For a telemanipulation set up, instead of connecting that port to a supervision controller, we can consider two identical systems and connect their supervision ports to the two extremes of a communication line, as schematically shown in Fig. 6.

Let us refer now to the one-dimensional teleoperator scheme shown in Fig. 5. In the scalar case, we have $Z = b \in \mathbb{R}^+$, and from (19) we obtain the usual scattering transformation with *force* and *velocity* as dual variables for a mechanical Hamiltonian system

$$\begin{cases} s^+ = \frac{1}{\sqrt{2b}}(F + bv) \\ s^- = \frac{1}{\sqrt{2b}}(F - bv). \end{cases} \tag{27}$$

Considering Fig. 5, the Hamiltonian system of Fig. 3 can be now represented by the robot

$$\begin{pmatrix} \dot{x} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{p}{m} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} F_{ext} \\ F_c \end{pmatrix}$$
$$\begin{pmatrix} v_{ext} \\ v_c \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{p}{m} \end{pmatrix} \tag{28}$$

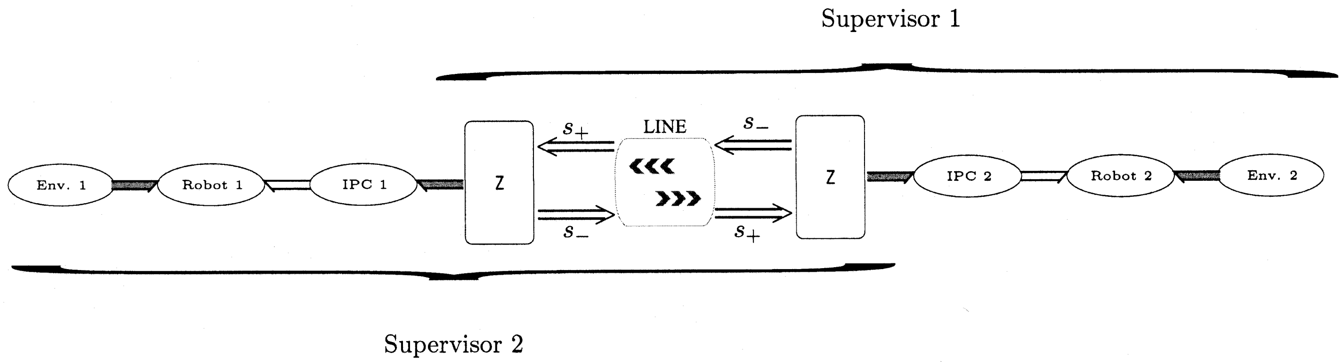

Fig. 6. IPC supervisor telemanipulation setting.

and the controller

$$\begin{pmatrix} \dot{\Delta x_c} \\ \dot{p_c} \\ \dot{\Delta x} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & (-b_v - b_l) & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} k_c \Delta x_c \\ \frac{p_c}{m_c} \\ k\Delta x \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ b_l & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ v_c \end{pmatrix}$$

$$\begin{pmatrix} F \\ -F_c \end{pmatrix} = \begin{pmatrix} -1 & b_l & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} k_c \Delta x_c \\ \frac{p_c}{m_c} \\ k\Delta x_c \end{pmatrix} - \begin{pmatrix} b_l & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v \\ v_c \end{pmatrix} \tag{29}$$

where Δx and Δx_c are the displacement of the springs k and k_c, respectively, p and p_c the momenta of the robot and controller, b_v is the *viscous* friction of the controller [20], and b_l is the damper related to the *line* that we have to choose in order to obtain the matching condition of *Principle 1*, and which corresponds to $B(x)$ in (21).

It can be seen that (29) is of the same form as (21), with, in this case, the Hamiltonian function given as

$$H_c(\Delta x, p_c, \Delta x_c) = \frac{1}{2}\left(\frac{p_c^2}{m_c} + k\Delta x^2 + k_c {\Delta x_c}^2\right).$$

Moreover, we have the two power ports (F, v) and $(-F_r, \dot{x})$, where the first one is used to connect to the transmission line and the second to connect to the robot with energy function

$$H_r(p) = \frac{1}{2m}p^2.$$

Hence, by choosing $b_l = b$, there will not be any reflected wave from master to slave system and vice versa. Hereafter, experimental results are shown to explain the behavior of a real master-slave system using this physical controller.

VI. Spatial Telemanipulation

The presented theory can be used to passively implement spatial telemanipulation. With this, it is meant that the developed theory is well posed in a coordinate-free setting, and therefore, it is possible to transmit not only scalar variables or numerical arrays, but geometric tensors like twist and wrenches. A situation which will be illustrated in an example is the case in which

$$V = se(3) \times \cdots \times se(3).$$

The reader should be aware that elements of $se(3)$ are not just numerical arrays, but tensors that can be given a geometrical interpretation of a screw [21].

In such a situation, the transmitted power variables will be a set of twists and their dual wrenches. To keep decoupled twists during transmission, the chosen line impedance should be of the form

$$Z = \begin{pmatrix} Z_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & Z_{n+1} \end{pmatrix}. \tag{30}$$

This can be used in complex telemanipulation systems using IPC techniques like the ones presented in [2], in which variations of geometric spring's length is controlled by a twist, an element in $se(3)$. The importance of the presented theory becomes relevant in these kind of applications. It is possible to transmit in an invariant way all that geometrical information which characterizes the control of complex spatial systems, as is done in [22].

A. Nontrivial Spatial Example

In order to illustrate in what kind of complex situations the presented techniques could be used, consider a master and slave system in which two identical robotic hands are available at the master and slave side, respectively. Suppose that the inertial properties of the hands are comparable, but the master hand construction is made in such a way that it can be used as a master exoskeleton. Suppose, furthermore, that both hands are fully actuated.

In such a situation, we can implement an IPC which would be mechanically equivalent to a set of spatial elements as shown in Fig. 7. Once the virtual dynamic system is simulated in real time, the elastic wrenches generated by the virtual springs can be translated into motor torque commands just by using the transpose of the geometric Jacobian of the hand.

As explained in [2], the element in the middle is called *virtual object*, and its function is to coordinate the fingers and to

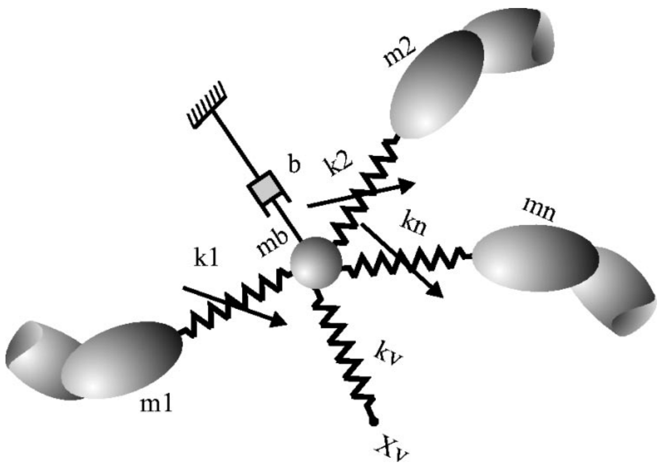

Fig. 7. Basic idea of the grasping IPC.

dissipate equivalent mechanical energy when it moves through the virtual damper attached to it.

The virtual object is attached to the finger tips by spatial springs which do have geometrical properties like center of stiffness and anisotropical elasticity. Furthermore, it has been shown in [23] that it is possible to define spatial springs with four power ports: two for attaching the spring to two inertial elements, one to control the rest spring configuration, and one to control the location of the center of stiffness and the orientational properties of the spring. All these ports are geometric and characterized by a twist-wrench pair.

Furthermore, there is an extra spring which is attached on one side to the virtual object and the other side can be used to move the complete hand in space (x_v in the picture).

The previously described IPC state can, therefore, be influenced through the transmission line using $2n+1$ power ports, where n is the number of fingers of the hands and a possible impedance structure, which can be used to transmit this geometrical information, which would be of the form reported in (30)

$$Z_i = \begin{pmatrix} Z_i^c & 0 \\ 0 & Z_i^v \end{pmatrix}, \qquad i = 0 \dots n; \quad Z_n = Z_v$$

and Z_i^c is the impedance used to code the center-of-compliance port of the finger i, Z_i^v its change in rest length, and Z_v the one used to code the port corresponding to the free side of the spring connected to the virtual object.

In the previously sketched situation, the sequence of events describing a telemanipulation session using the exoskeleton can be simply described as follows. First, the master opens the hand and the spatial springs lengths are reflected on the corresponding power ports attached to the transmission line. The corresponding slave springs will vary, reflecting the master situation. The master moves the hand toward an object to be grasped, and by doing so, it loads the last spring connected to the virtual object, and this situation will be reflected on the slave through the power port.

In all the steps described previously, all information is sent in a geometrically consistent way, thanks to the coordinate invariance of the presented techniques. For more details, the reader is referred to [22].

VII. CONCLUSIONS

In this paper, a general setting for telemanipulation of port-Hamiltonian systems and, therefore, any explicit physical system has been presented. A new system theoretical condition has been introduced, which can be used to test whether proper matching is taking place. A possible measure of matching has been also introduced.

It has been shown that the standard form of explicit port-controlled Hamiltonian systems is not general enough to obtain matching and it must be extended by a feed-throw term. This can be shown more generally using directly a network structure as shown in Section V-A.

The presented theory is important for the implementation of geometrical telemanipulation where the vector space $se(3)$ is used. It is known in the literature [24] that $se(3)$ does not have any intrinsically defined, strictly positive-definite metric and, therefore, any possible choice of scattering decomposition is dependent on an additionally chosen metric as shown in Section II.

The major important contribution is the formalization of an invariant way which allows transmitting geometrical entities along a transmission line as real geometrical tensors and *not* as an array of scalar components.

Another paper will present an extension of the presented results for time-varying time delays. Preliminary results in this direction have been presented in [25].

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